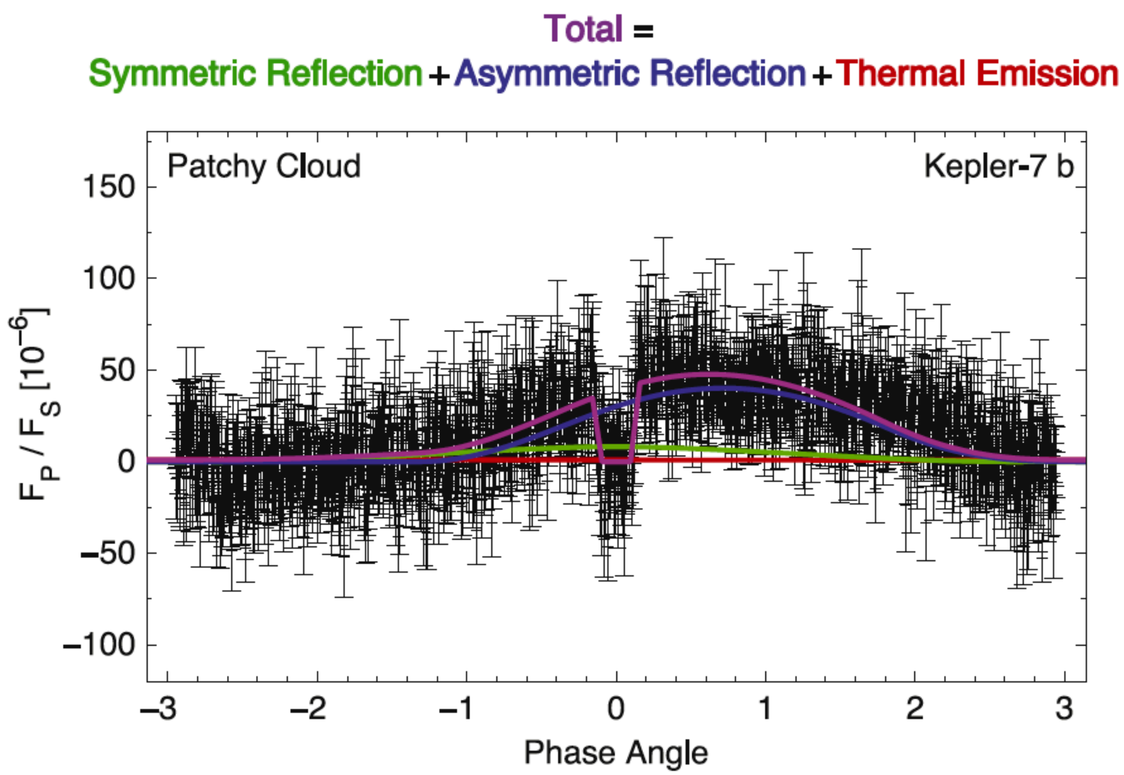
How Do We Intepret Phase Curves?   Standard Practice



Hu et al. (2015, ApJ, 802, 51)

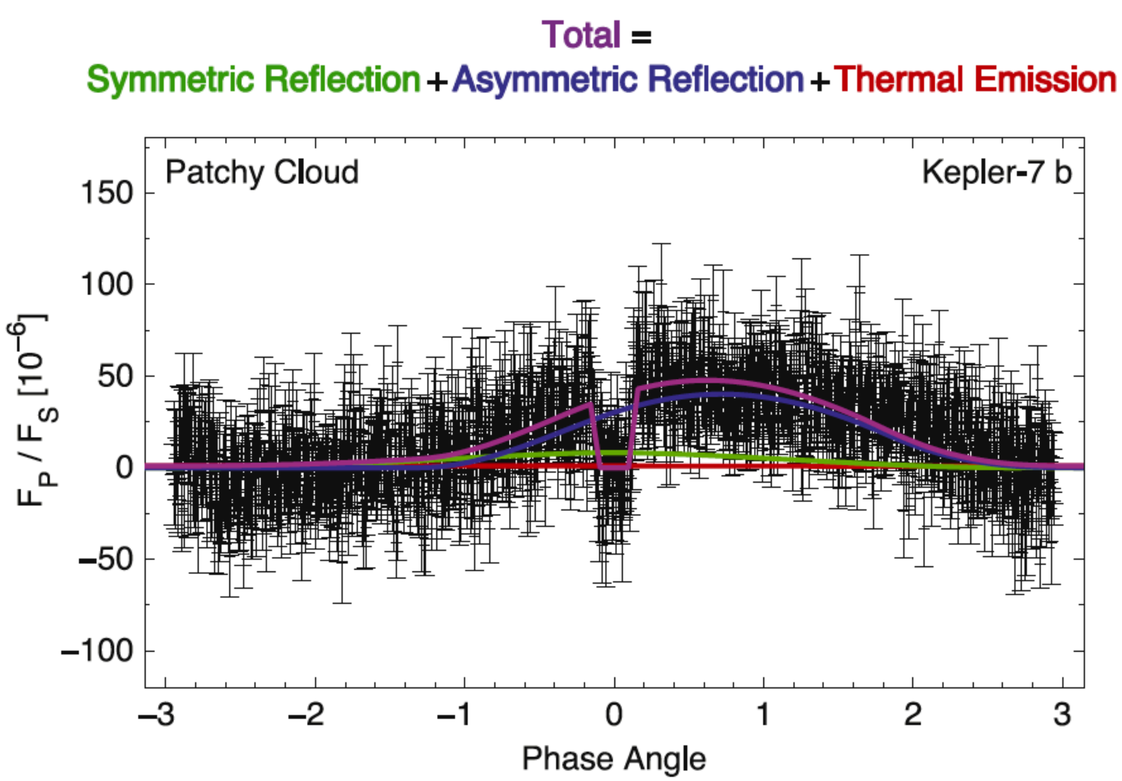
thermal emission

Dayside temperature
Nightside temperature
Redistribution efficiency
Bond albedo

reflected light

Geometric albedo

How Do We Intepret Phase Curves? New Approach



Hu et al. (2015, ApJ, 802, 51)

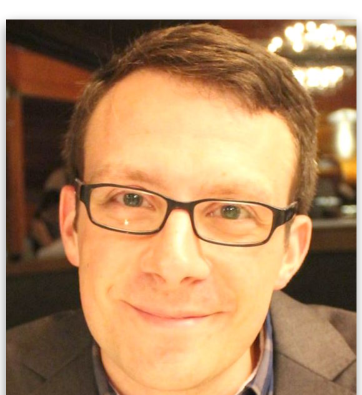
thermal emission

Dayside temperature
Nightside temperature
Redistribution efficiency
Bond albedo

Temperature map
Background temperature
(efficiency & Bond albedo)
Phase shift

reflected light

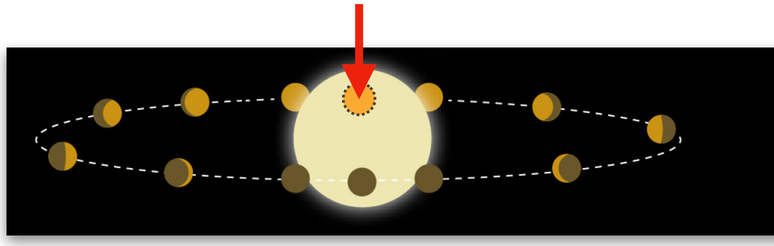
Single-scattering albedo
Scattering asymmetry factor
Geometric albedo
Spherical albedo
Phase integral

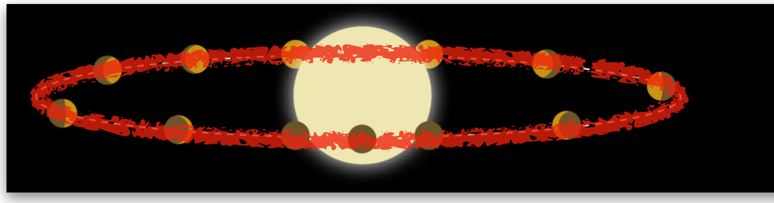


Code originally written by Brett Morris
(https://github.com/bmorris3/kelp)

Jargon: reflected light component

Quantity	Definition
Geometric albedo	Albedo at zero phase angle (superior conjunction)
Spherical albedo	Albedo over all phase angles
Bond albedo	Spherical albedo over all wavelengths
Phase integral	Ratio of spherical to geometric albedo
Single-scattering albedo	Fraction of light scattered in a single event





$$A_B = \frac{\int A_s I_\star \, d\lambda}{\int I_\star \, d\lambda}$$

$$q = \frac{A_s}{A_g} = 2\int_0^\pi \Psi \sin\alpha \, d\alpha$$

$$\omega$$

THE ASTROPHYSICAL JOURNAL, 678: L129–L132, 2008 May 10

INVERTING PHASE FUNCTIONS TO MAP EXOPLANETS

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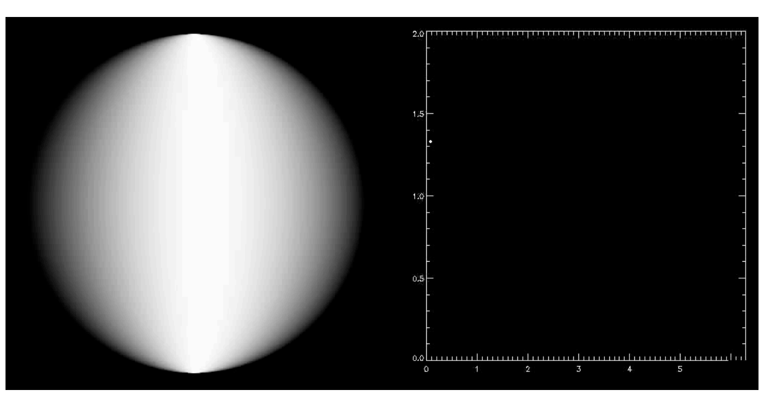
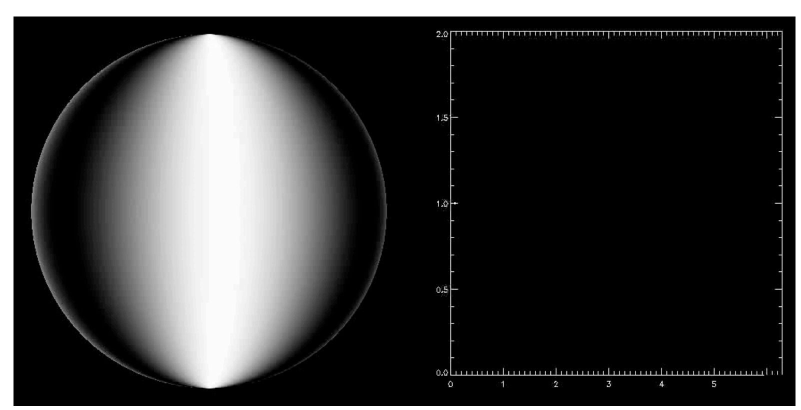
Received 2008 February 14; accepted 2008 March 25; published 2008 April 15

- Phase curve is a convolution between temperature map and viewing geometry.
- If temperature map is described by a series of sines and cosines (with no physical meaning), then one may write down a second series (also of sines and cosines) for the phase curve.
- Deals only with thermal emission.
- Since about half the information is lost in this convolution, phase curves are described by at most ~5 parameters.



Even modes contribute to the phase curve……



but odd modes are invisible!

Videos courtesy of Eric Agol

THE EIGENFUNCTIONS OF LAPLACE'S TIDAL EQUATIONS OVER A SPHERE

By M. S. LONGUET-HIGGINS, F.R.S.
National Institute of Oceanography, England and Scripps Institution of Oceanography, La Jolla, California

Source: *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 262, No. 1132, (Feb. 29, 1968), pp. 511-607
Published by: The Royal Society
Stable URL: http://www.jstor.org/stable/73582

- Solved "shallow water equations" (Laplace's tidal equation) on rotating, frictionless sphere in pure hydro limit.

THE ASTROPHYSICAL JOURNAL, 703:1819–1831, 2009 October 1 doi:10.1088/0004-637X/703/2/1819

MAGNETOHYDRODYNAMIC SHALLOW WATER WAVES: LINEAR ANALYSIS

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Received 2008 December 10; accepted 2009 August 14; published 2009 September 14

- Added magnetic fields to Longuet-Higgins's (1968) treatment, in order to understand Type I X-ray bursts from **neutron stars**.

- Discovered so-called "magnetostrophic mode".

The Astrophysical Journal Supplement Series, 213:27 (35pp), 2014 August doi:10.1088/0067-0049/213/2/27

ANALYTICAL MODELS OF EXOPLANETARY ATMOSPHERES. I. ATMOSPHERIC DYNAMICS VIA THE SHALLOW WATER SYSTEM

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Received 2014 January 29; accepted 2014 June 2; published 2014 July 18

- Generalised previous work to treat rotating, heated, magnetised sphere with friction, i.e., magnetohydrodynamics (MHD).
- In limit of static sphere in hydro limit, recover spherical harmonics.

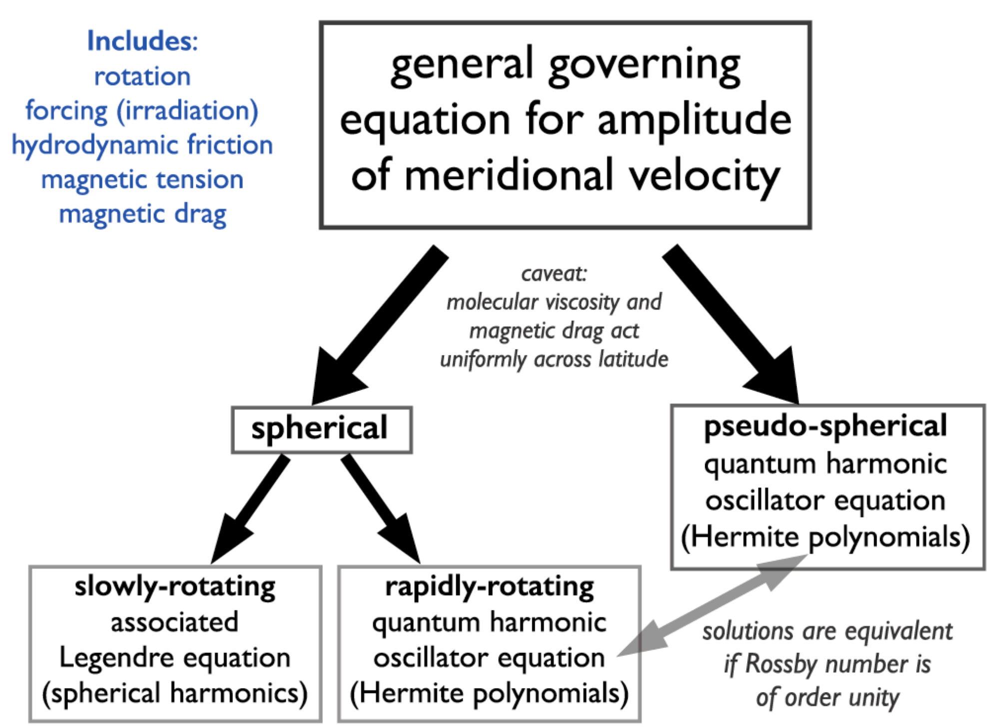


Figure 1. Schematic describing the key governing equation in shallow water systems, both on the equatorial β-plane (pseudo-spherical geometry) and in full spherical geometry. The key quantity to solve for is the meridional (north–south) velocity, from which the zonal (east–west) velocity, shallow water height perturbation, and magnetic field perturbations straightforwardly follow.

Table 2
Comparison to Previous Analytical Work

Reference	Spherical Geometry?	HD: Forcing+Friction?	MHD: Free?	MHD: Forcing+Friction?
Matsuno (1966)	N	Y	N	N
Lindzen (1967)	N	Y	N	N
Longuet-Higgins (1968)	Y	N	N	N
Gill (1980)	N	Y	N	N
Spitkovsky et al. (2002)	N	Y	N	N
Holton (2004)	N	N	N	N
Kundu & Cohen (2004)	N	N	N	N
Vallis (2006)	N	N	N	N
Zaqarashvili et al. (2007)	Y	N	Y	N
Heng & Spitkovsky (2009)	Y	N	Y	N
Showman & Polvani (2011)	N	Y	N	N
Heng & Workman (current work)	Y	Y	Y	Y

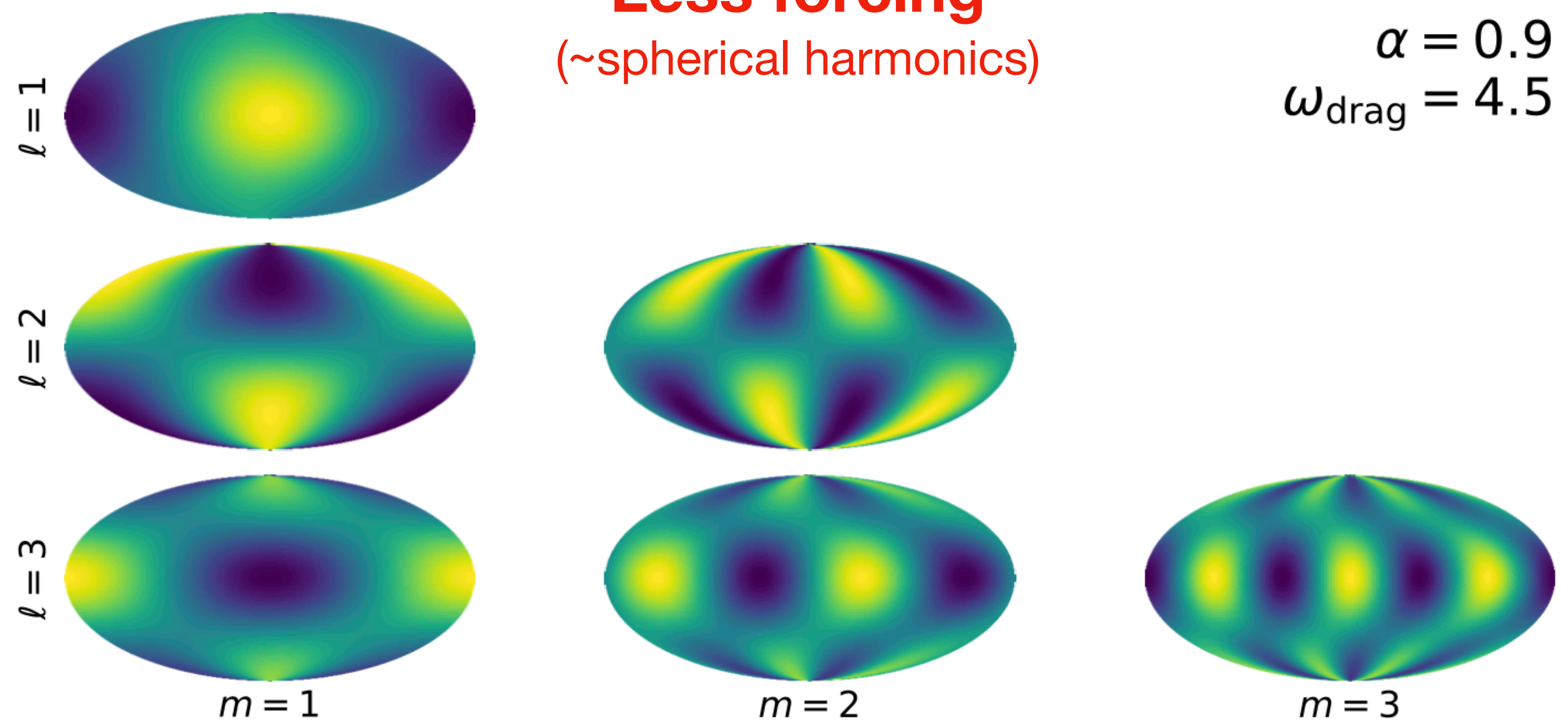
Note. HD: hydrodynamic. MHD: magnetohydrodynamic.

“Generalised spherical harmonic basis functions”
(parabolic cylinder functions)

Rotating, heated sphere with friction/drag
(Heng & Workman 2014, ApJS, 213, 27)

Less forcing
(~spherical harmonics)

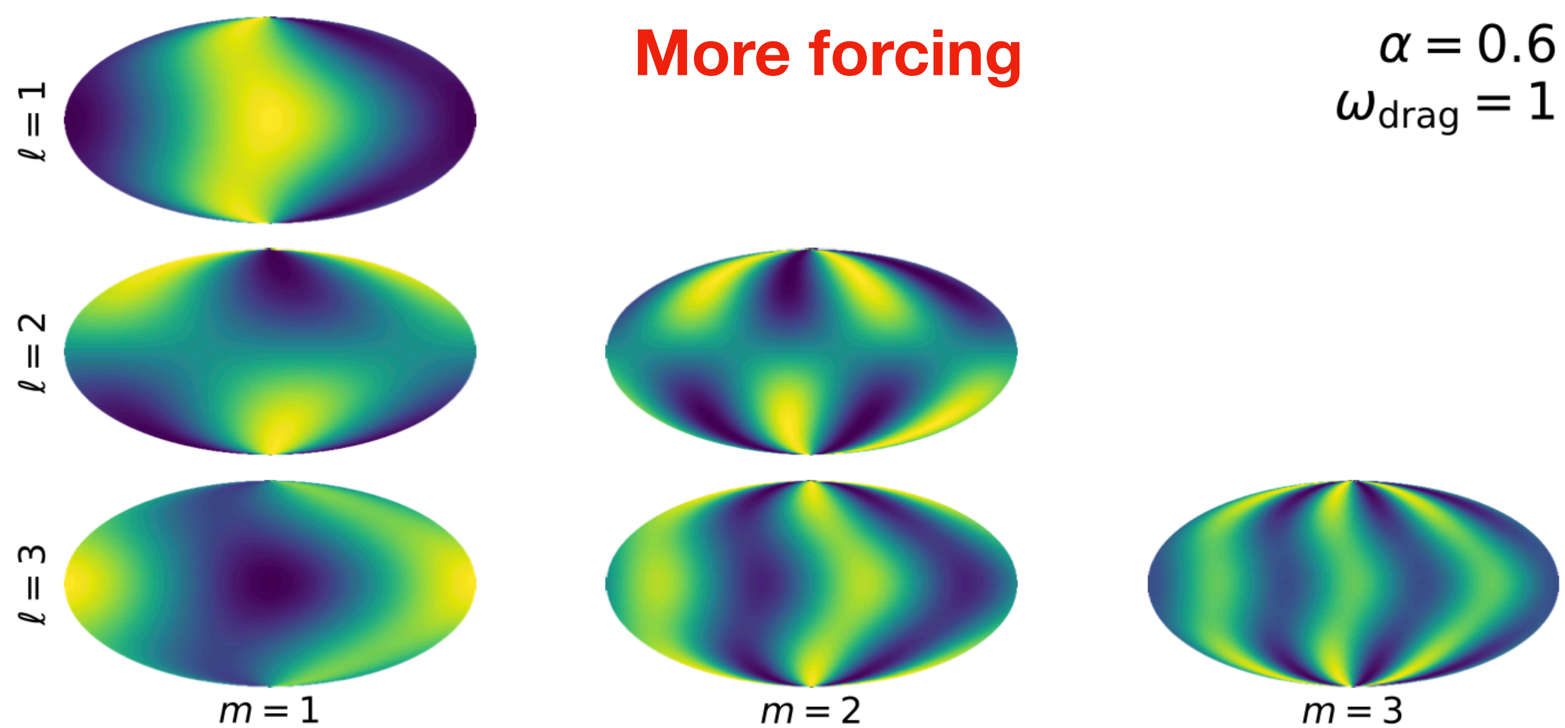
$\alpha = 0.9$
$\omega_{\rm drag} = 4.5$



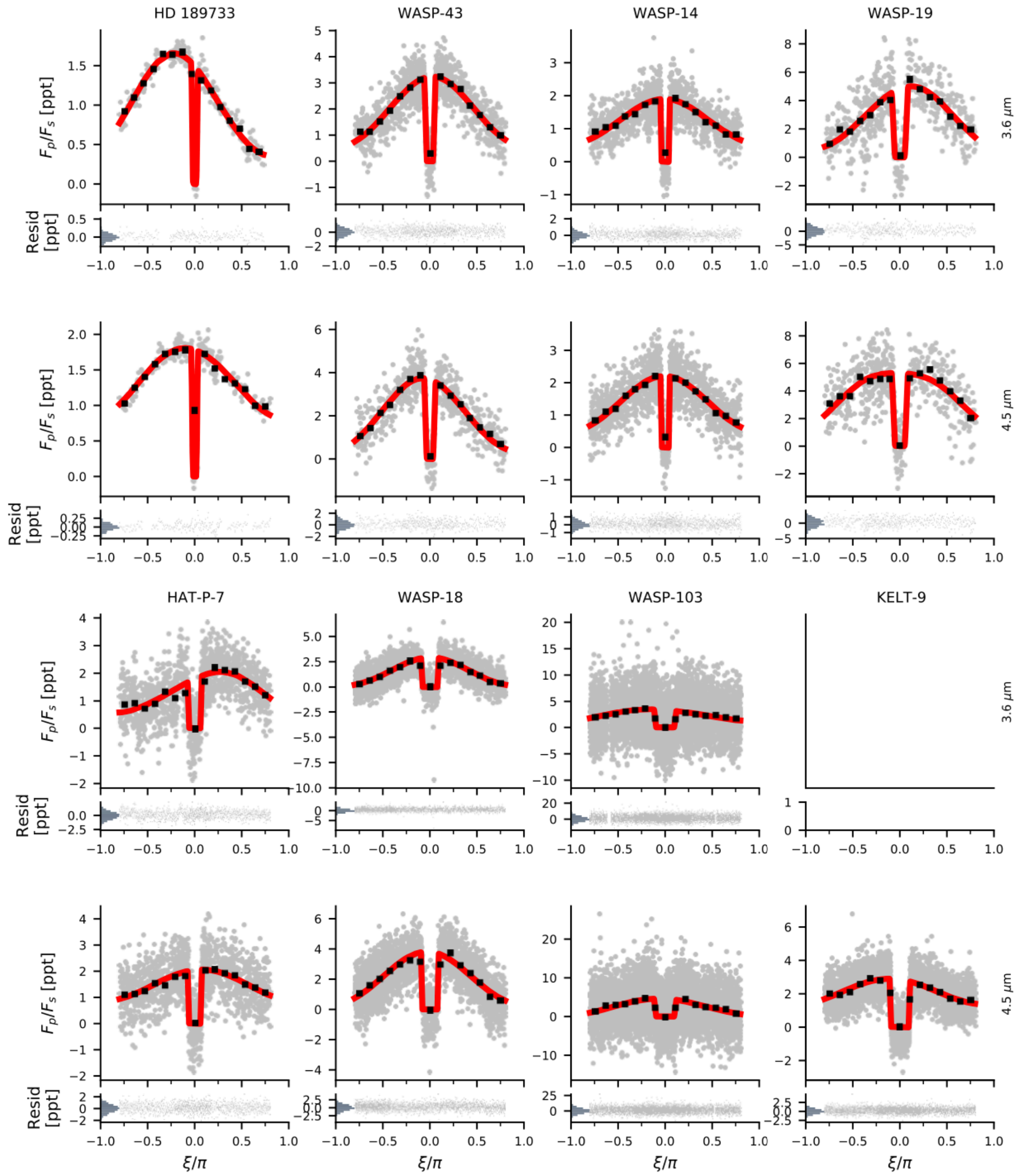
Advantages:
1. Parameters are physical
2. Much better than fitting ad hoc sines and cosines
3. Less parameters needed

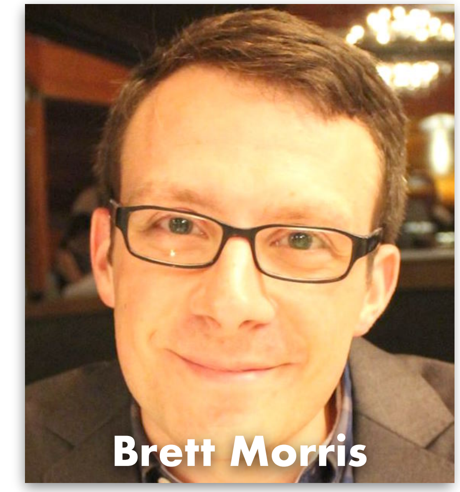
More forcing

$\alpha = 0.6$
$\omega_{\rm drag} = 1$



Morris, Heng et al. (2021, A&A, in press; arXiv:2110.11837)

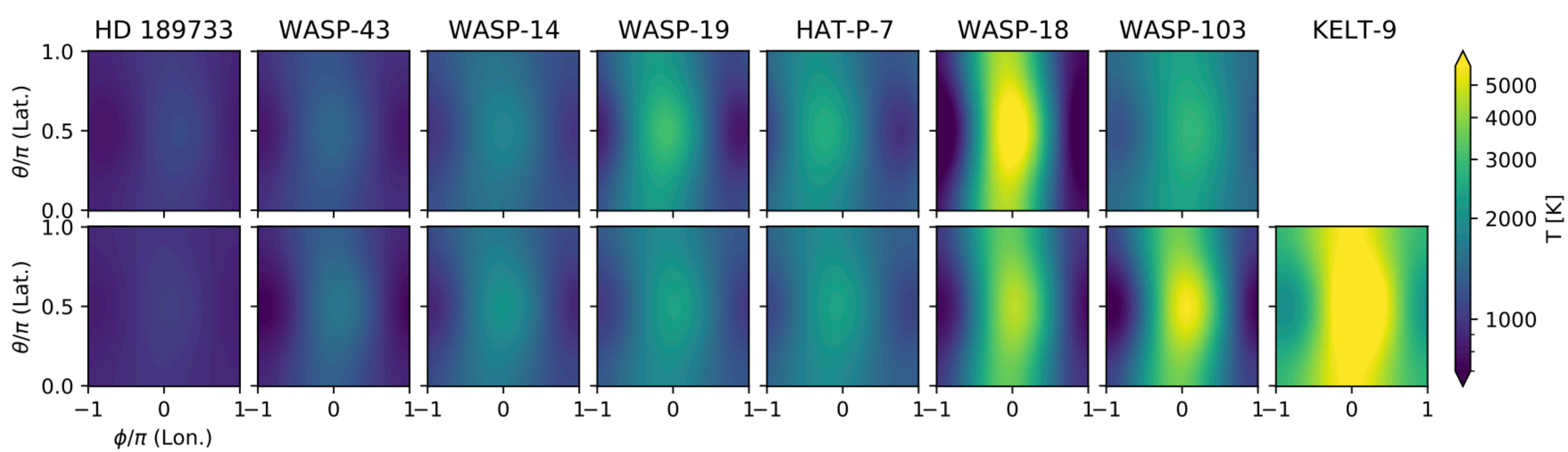


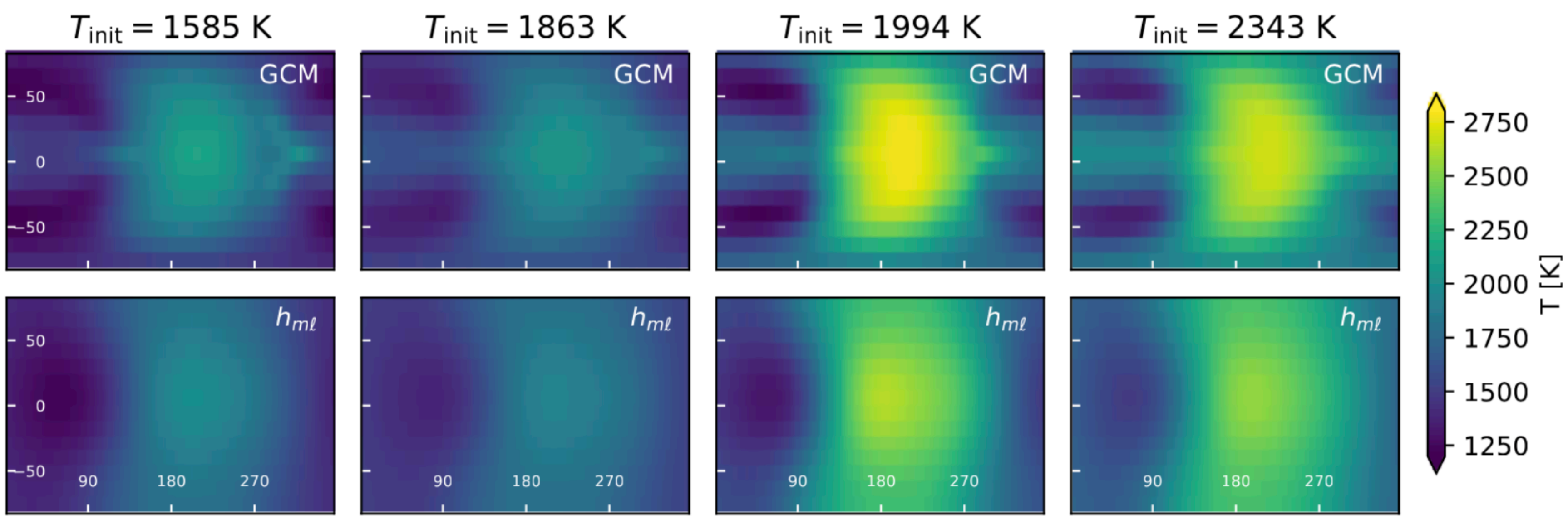


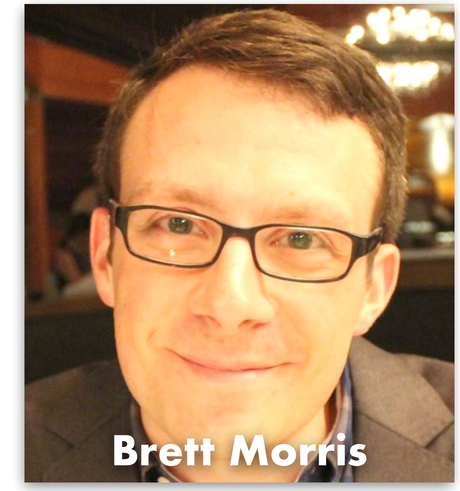
We can fit <u>all</u> of the Spitzer phase curves in existence with 3 parameters:

1. "Power" in main mode (C_{11})
2. Equilibrium temperature
3. Phase shift (longitude)

Morris, Heng et al. (2021, A&A, in press; arXiv:2110.11837)

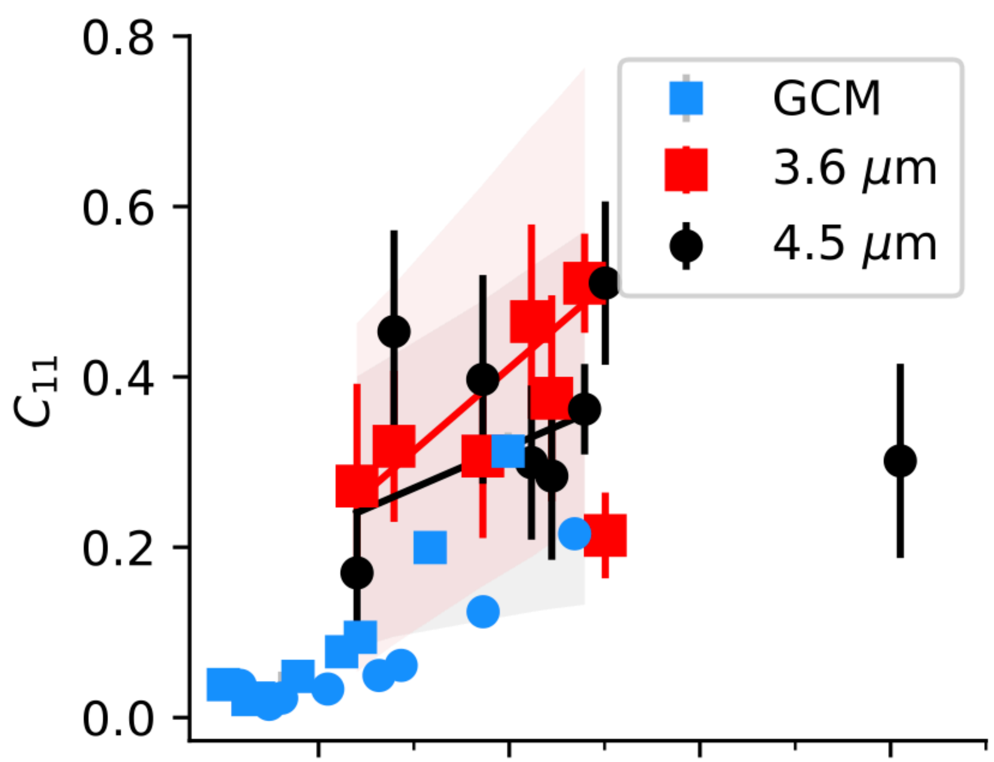


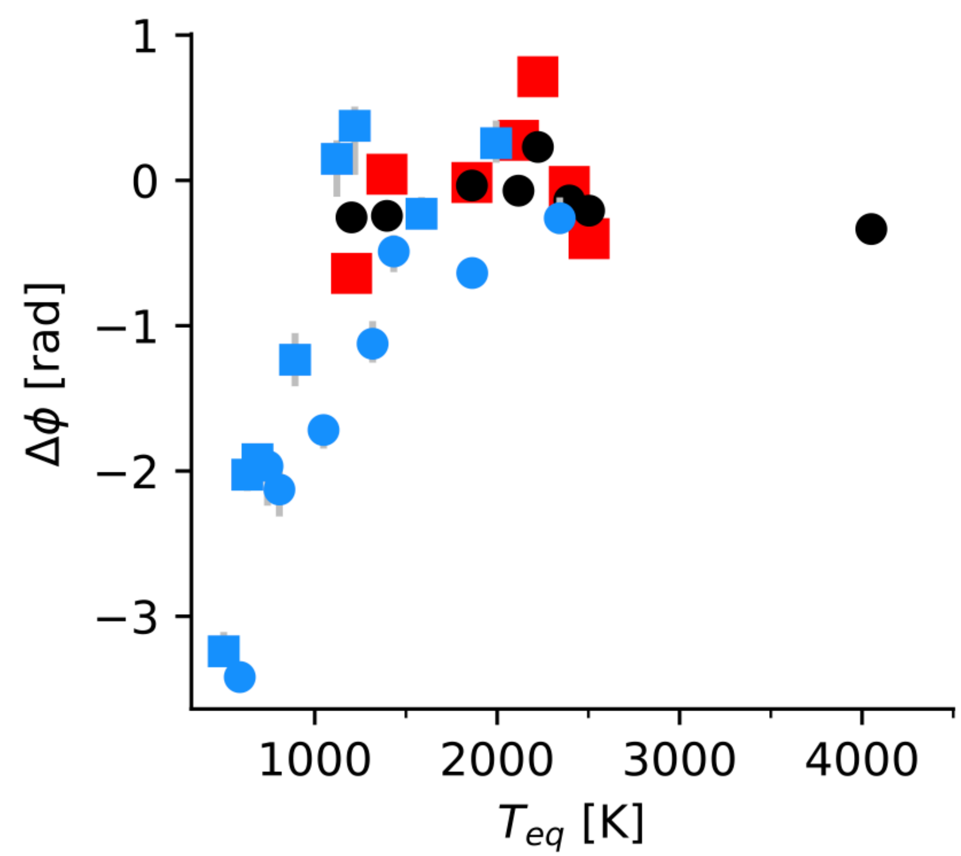




Morris, Heng et al. (2021, A&A, in press; arXiv:2110.11837)

Fitting simulated GCM temperature maps to calibrate latitudinal behaviour





Basis functions are "alphabet" for comparing GCMs versus observations <u>on same footing</u>

Reflected-light phase curves: quick summary

$$\frac{F}{F_\star} = A_g \left(\frac{R}{a}\right)^2 \Psi$$

geometric albedo
(**magnitude** of phase curve)

integral phase function
(**shape** of phase curve)

- ***Heng, Morris & Kitzmann (2021, Nature Astronomy, 5, 1001):***
  Ab initio solutions for geometric albedo and integral phase function for *any* reflection law (scattering phase function).

- Derived in limit of “semi-infinite atmosphere” (cf. Chandrasekhar) and assumes the fundamental scattering parameters are constant.

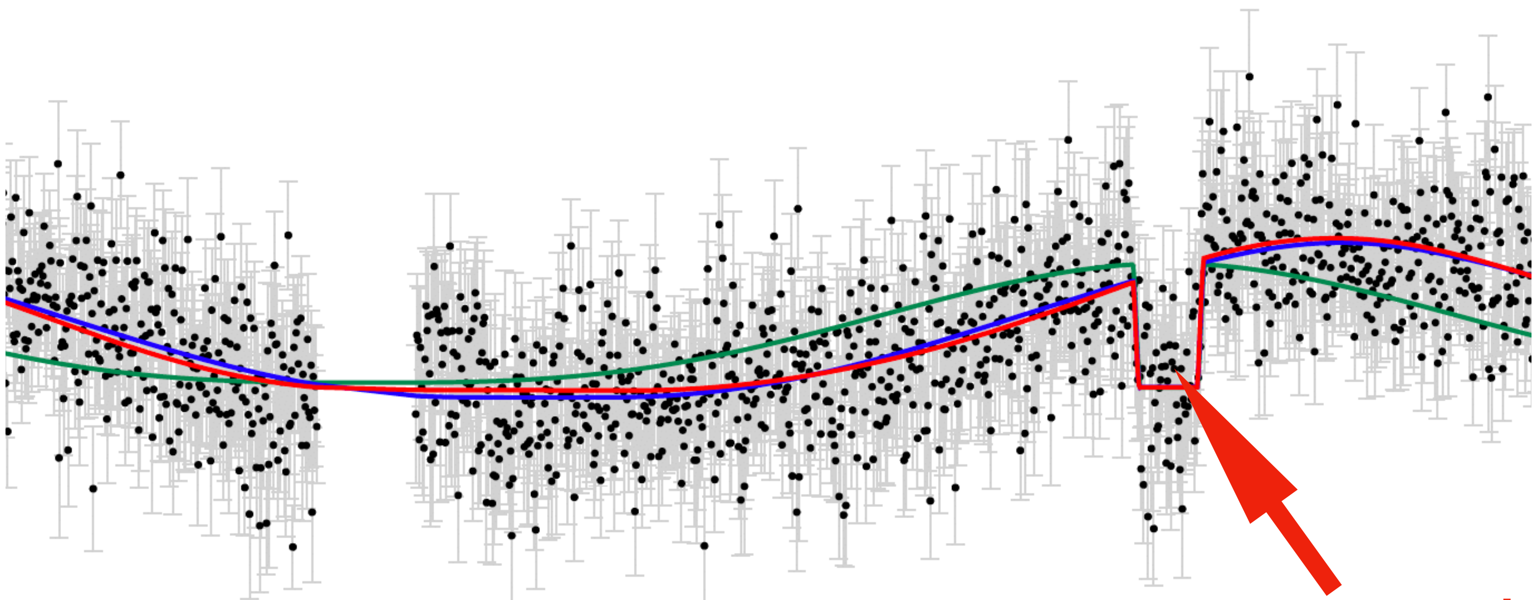
- Only one major approximation made: isotropic multiple scattering (cf. Hapke).

- But this is still a vast improvement over the Lambertian sphere solution.

Re-interpreting phase curves

Standard practice so far:
Fit shape and amplitude of phase curve independently using an ad hoc series of sinusodial functions

$A_g \Psi$



Demory et al. (2013, ApJL, 776, L25)

secondary eclipse yields geometric albedo

With our newly discovered solutions:
Shape and amplitude are determined by a set of fundamental parameters.

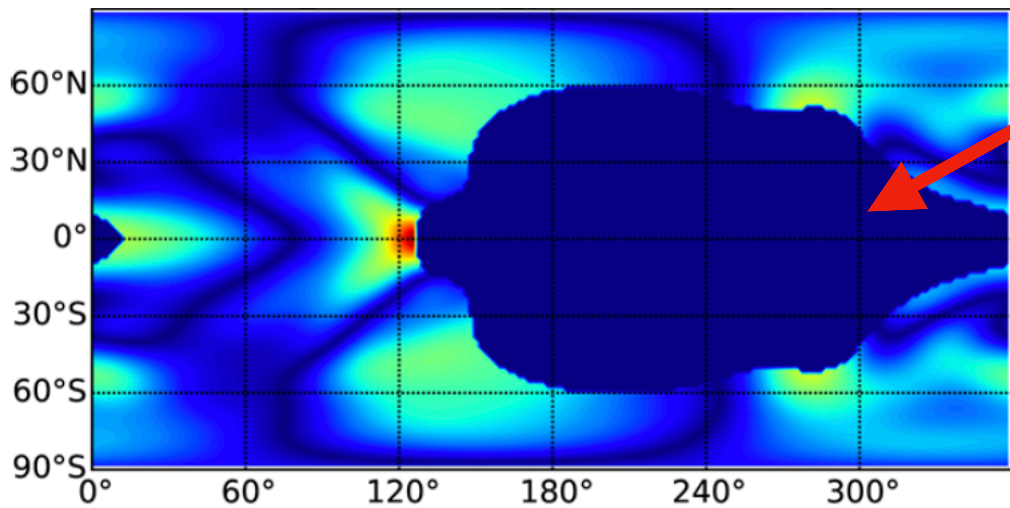
$$A_g(\omega, g)$$

$$\Psi(\omega, g)$$

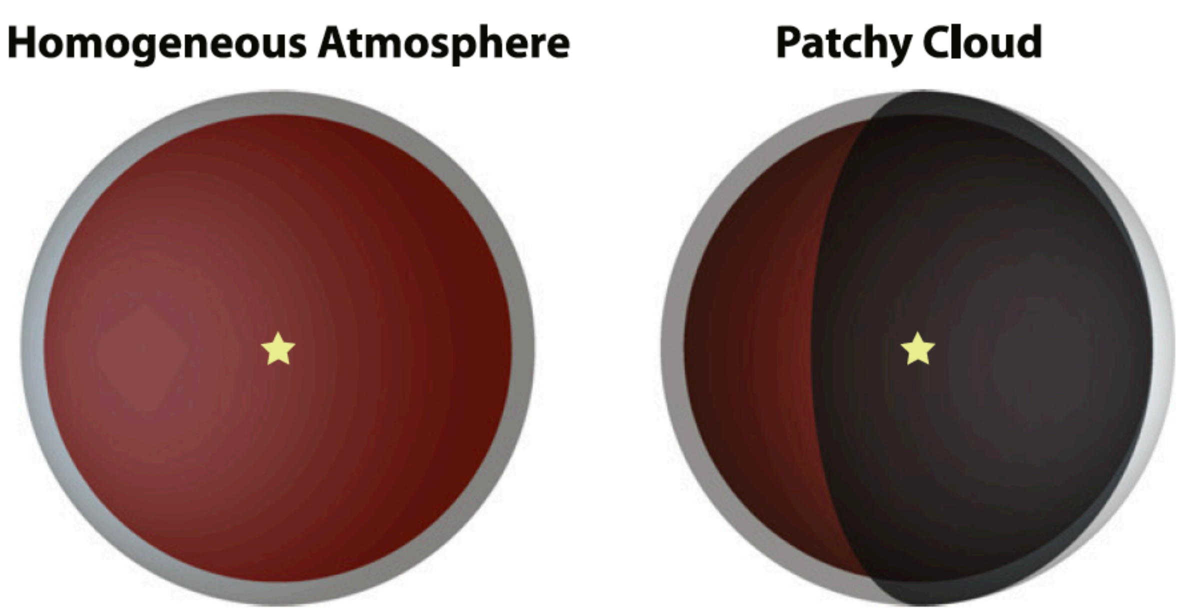
Heng, Morris & Kitzmann (2021, Nature Astronomy, 5, 1001)

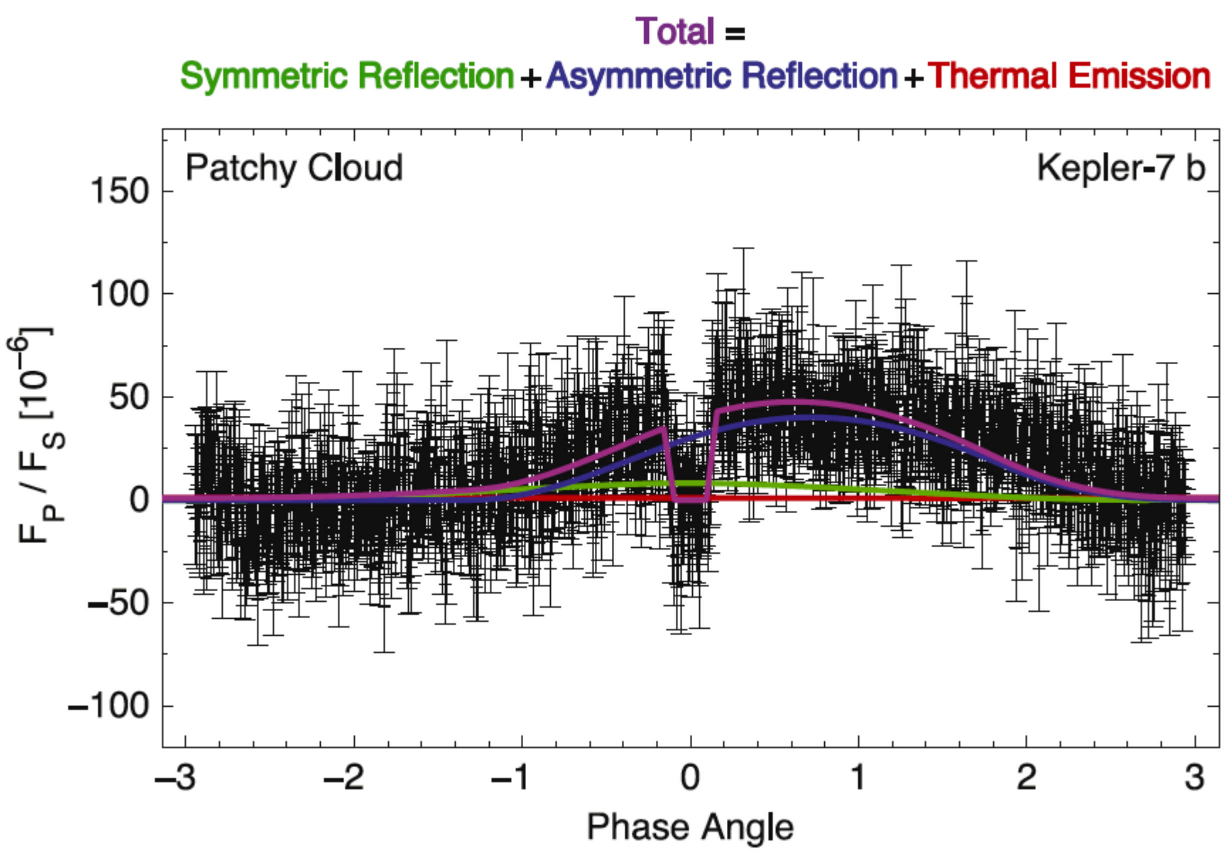
Westward peak offsets are direct evidence for inhomogeneous cloud/haze cover on exoplanets

Hotter regions are darker in reflected light, because aerosols/clouds cannot condense out!



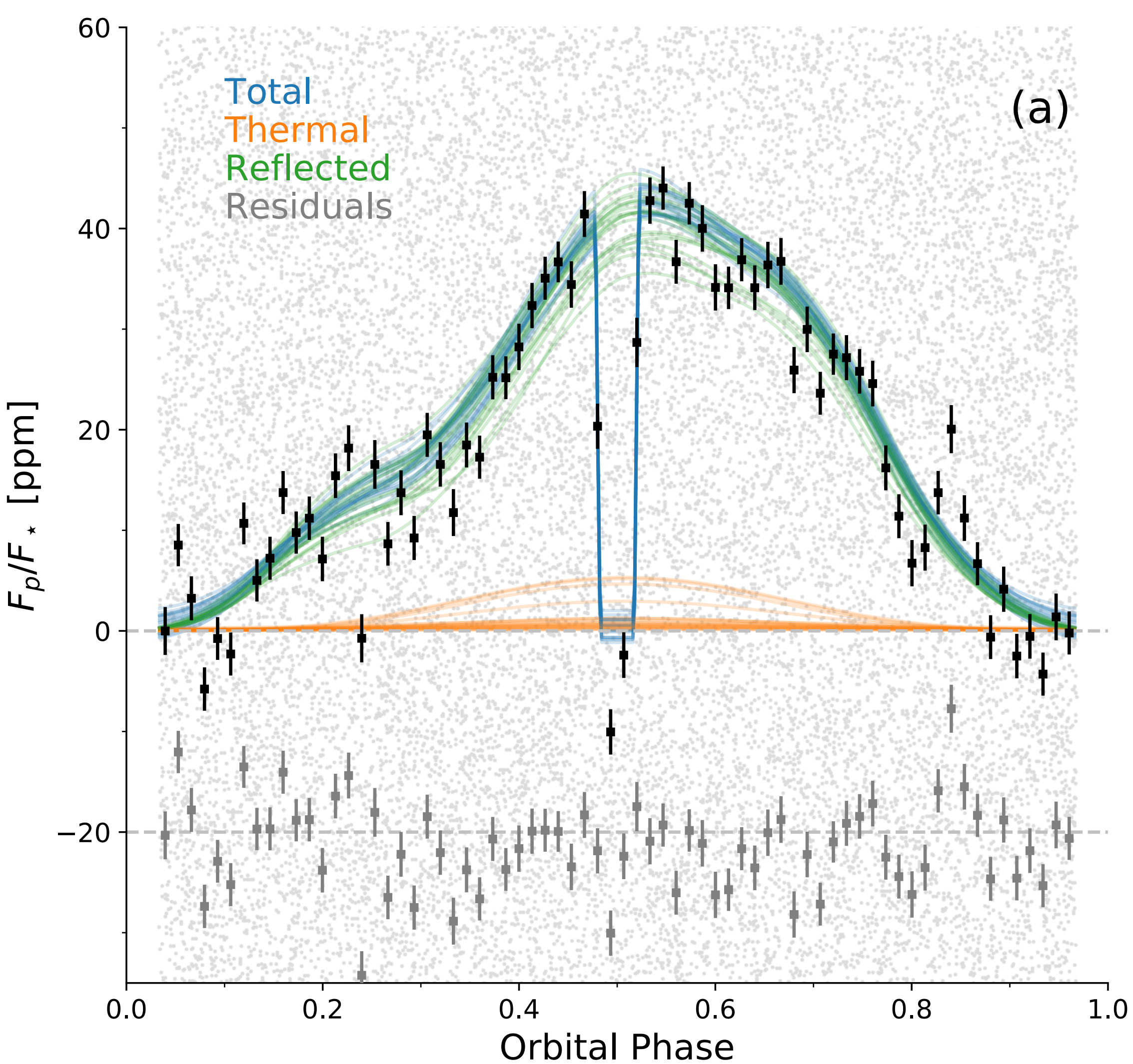
Oreshenko, Heng & Demory (2016, MNRAS, 457, 3420)

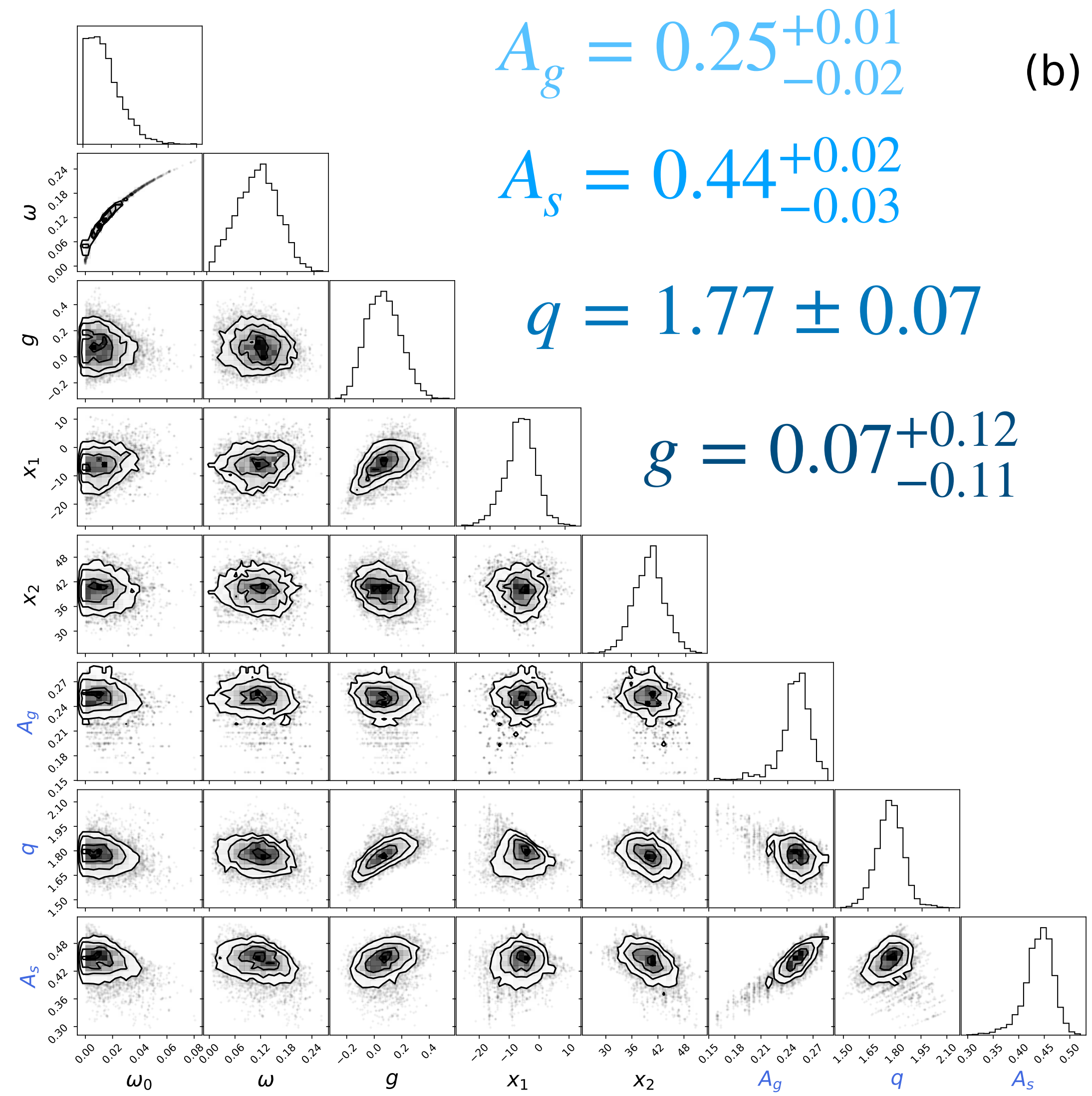


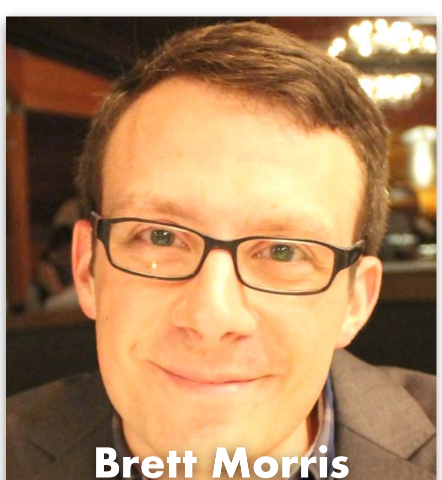


Hu et al. (2015, ApJ, 802, 51)

Re-interpretation of Kepler-7b phase curve





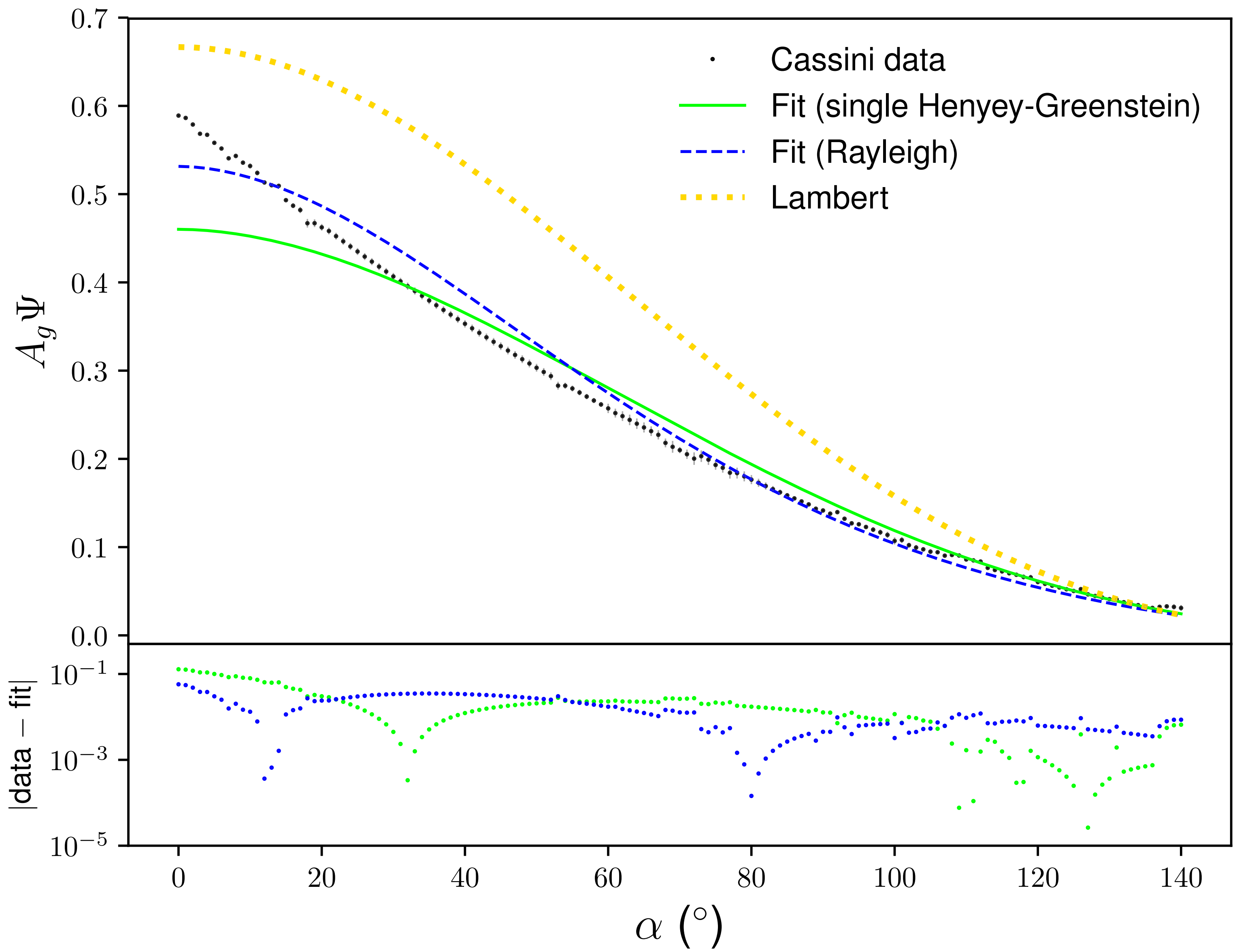


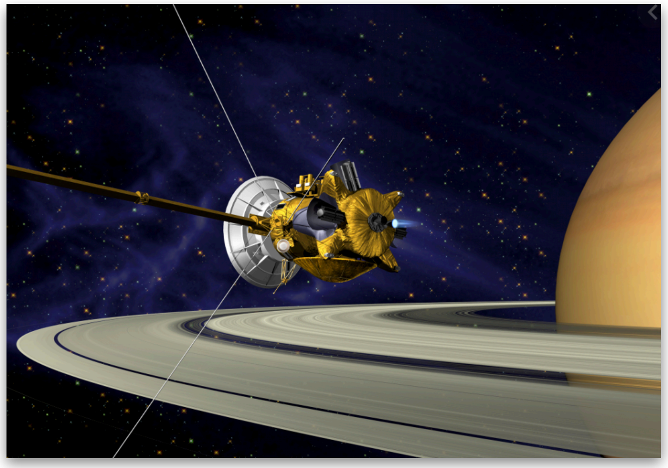
Spherical albedo and **phase integral** may be retrieved directly from reflected light phase curve of transiting exoplanet!

Aerosols are small and have a condensation temperature ~1600 K.

Heng, Morris & Kitzmann (2021, Nature Astronomy, 5, 1001)

A longstanding obstacle with interpreting Cassini phase curves of Jupiter



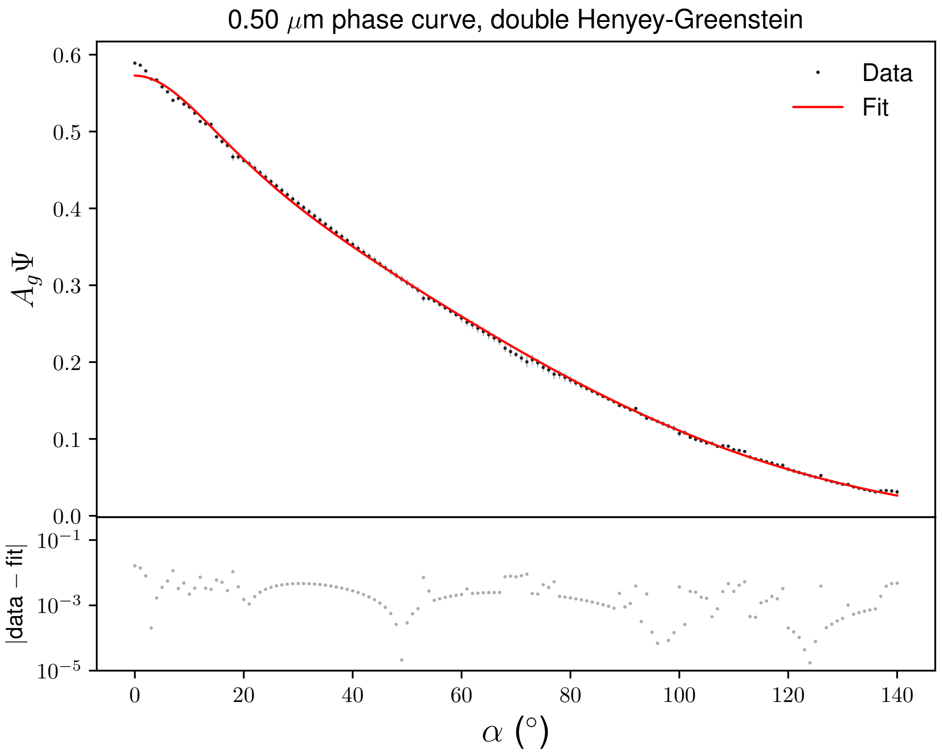


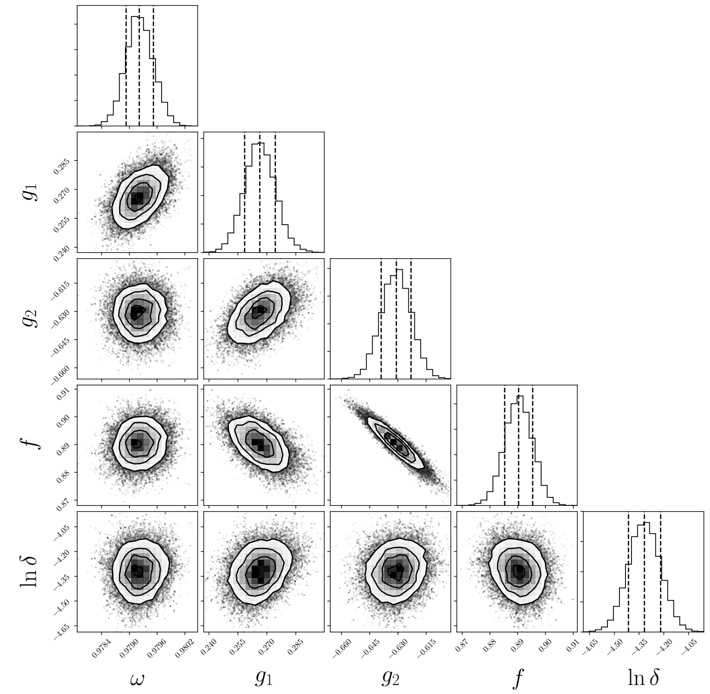
Cassini "millenium flyby" of Jupiter

Classic reflection laws provide poor fits, especially near the peak

Data have never been interpreted within a Bayesian framework

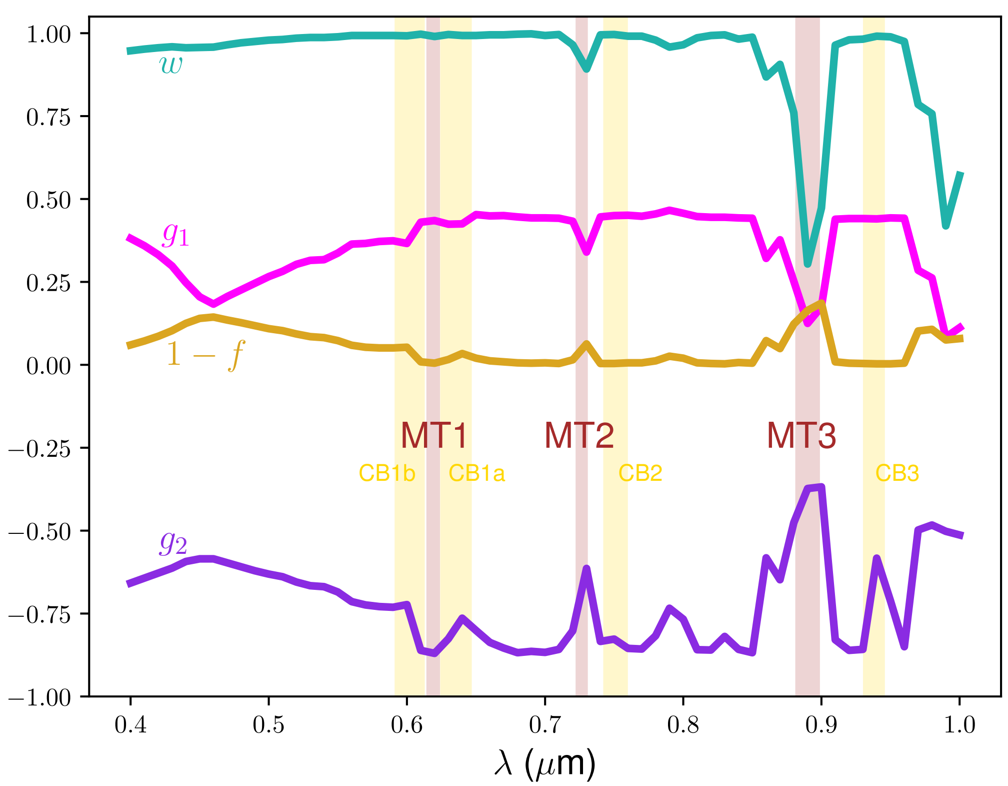
Heng & Li (2021, ApJL, 909, L20)





At each wavelength, fit for fundamental aerosol parameters

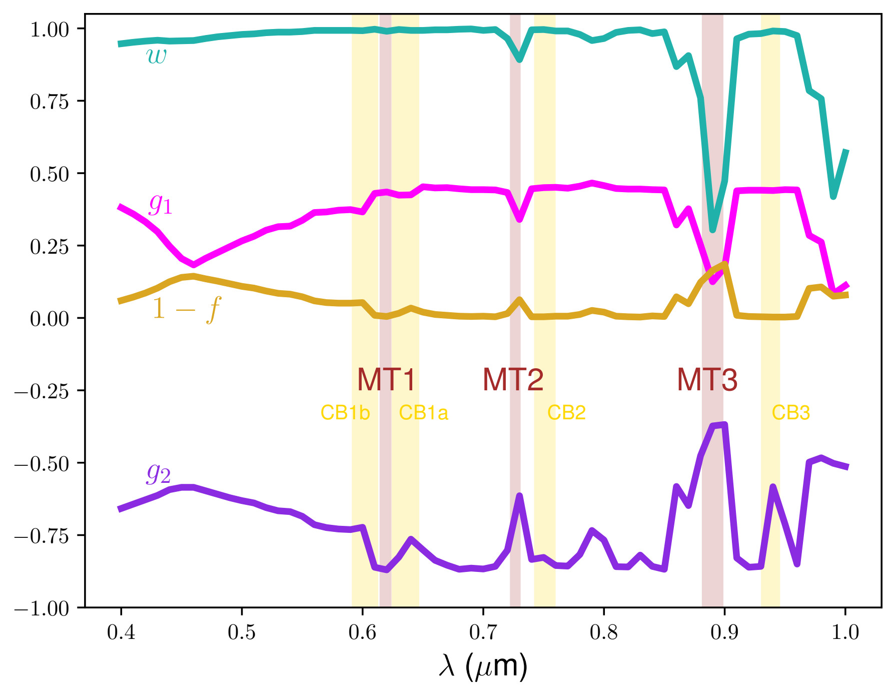
Perform fits to 61 Cassini phase curves
(0.4 to 1 micron)



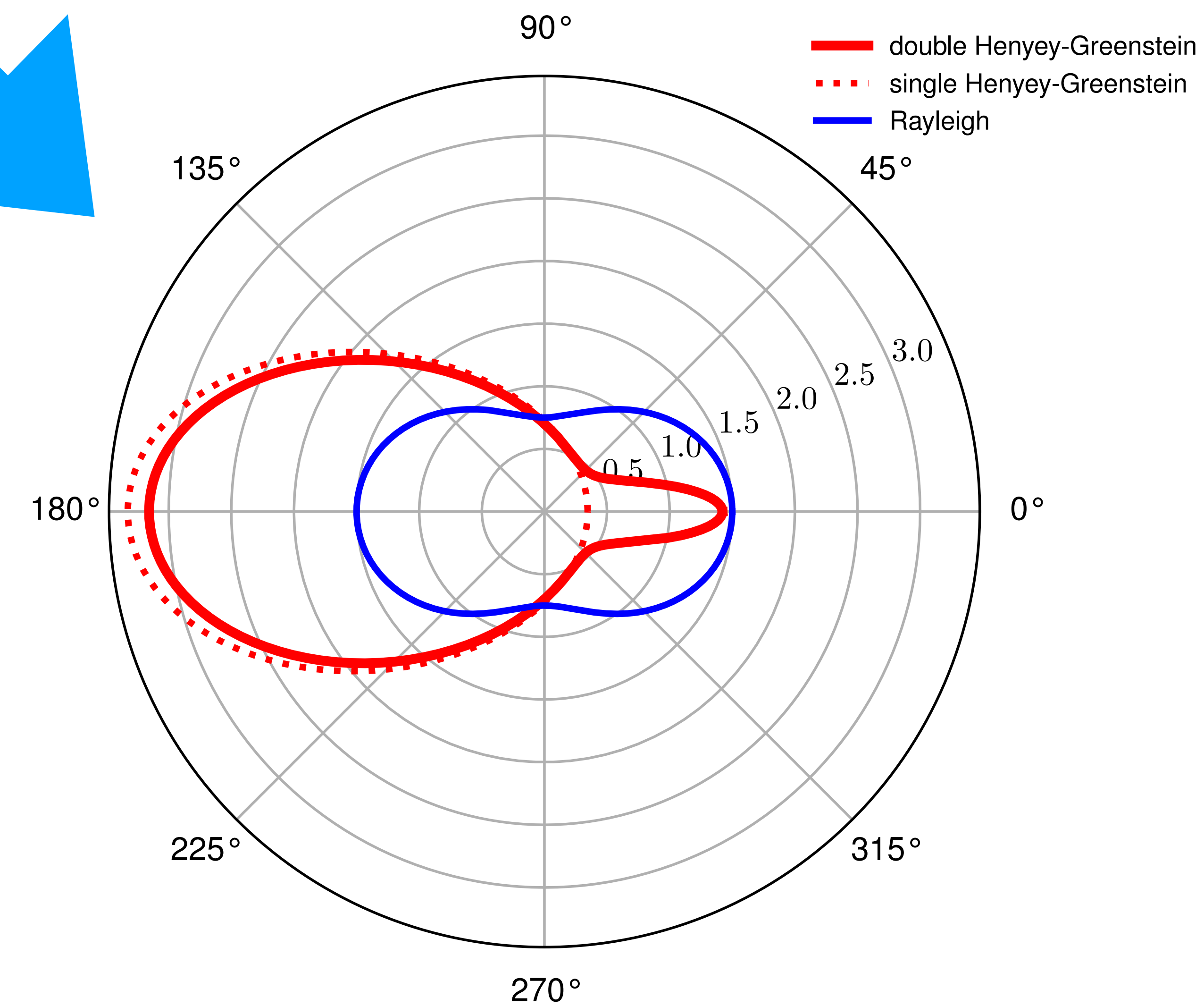
Construct aerosol properties as functions of wavelength!

Notice how reduction in scattering coincides with MT3 methane absorption band of Cassini's ISS instrument

Heng & Li (2021, ApJL, 909, L20)



Aerosol properties may be interpreted geometrically



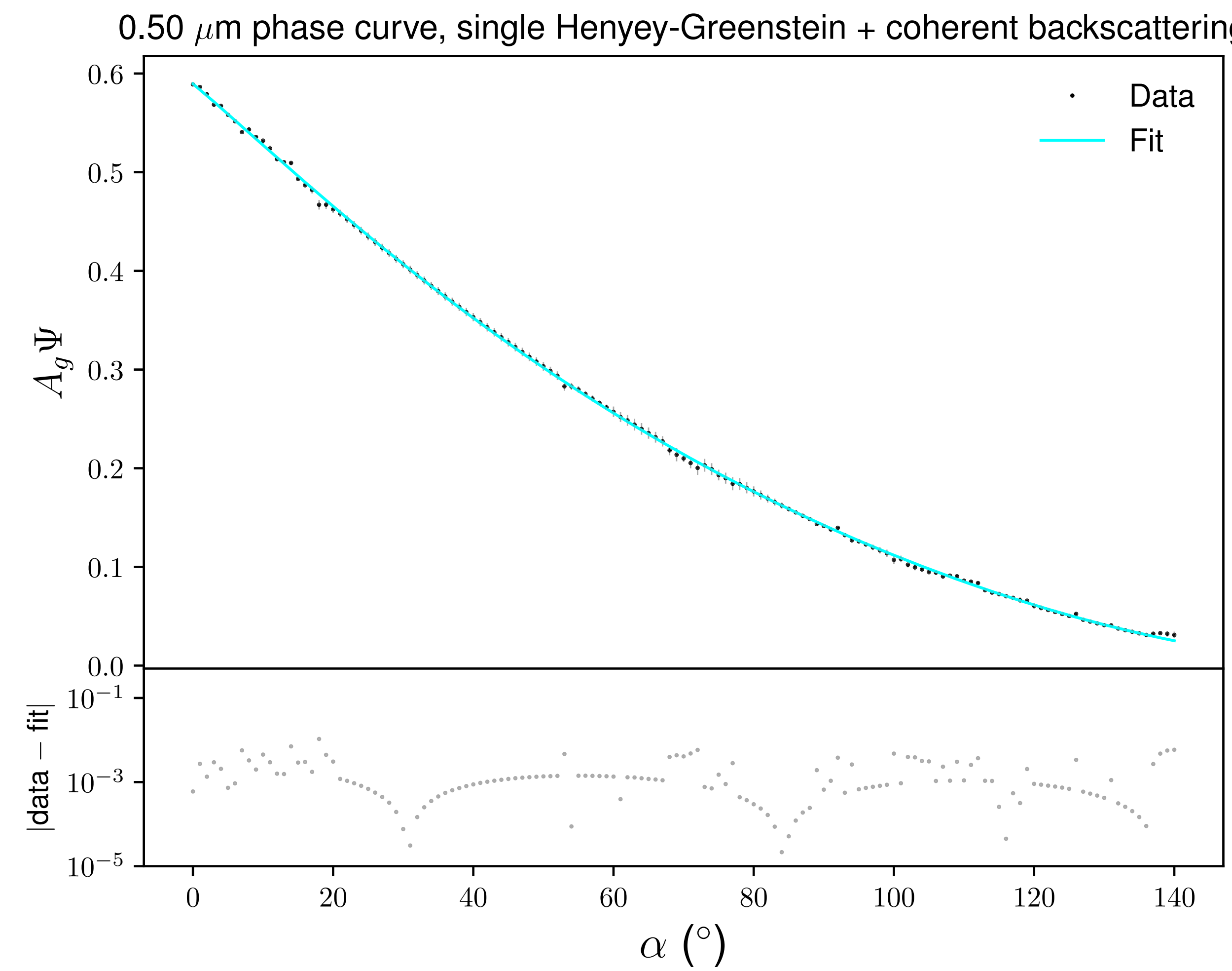
Aerosols in Jupiter's atmosphere:

1. Large, possibly irregular
2. Polydisperse
3. Inconsistent with Rayleigh scattering
4. Multiple scattering is important
5. Backscattering lobe
6. Of unknown chemical composition

Heng & Li (2021, ApJL, 909, L20)

Are we witnessing coherent backscattering in the Jovian atmosphere?

Coherent backscattering:
multiple scattering in non-uniform medium leading to constructive interference of light



Large body of literature exists (e.g. Hapke)

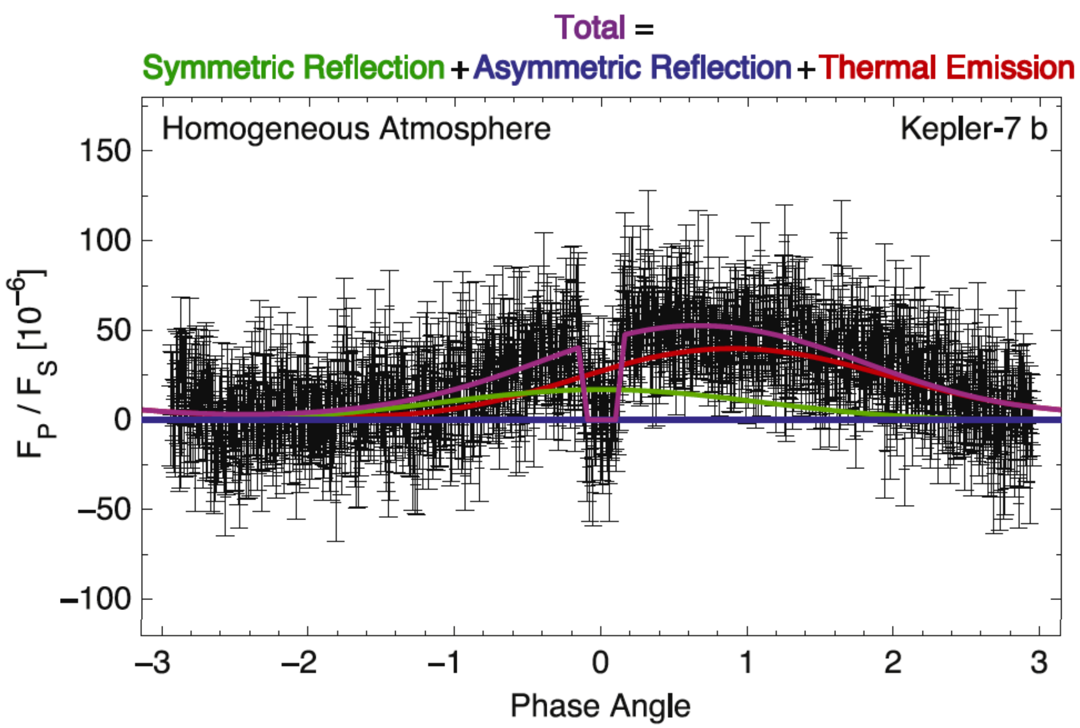
Has been cited as one of the explanations for cuspy phase curves, but this is usually for rocky bodies with surfaces and regolith

Remains to be proven for Jupiter

5-parameter fit with 2 parameters to describe coherent backscattering seems to work really well

Heng & Li (2021, ApJL, 909, L20)

Epilogue: what is kelp?



Hu et al. (2015, ApJ, 802, 51)

thermal emission

Uses generalised spherical harmonic basis functions of *Heng & Workman (2014)* to fit temperature map

Morris, Heng et al. (2021, A&A, in press; arXiv:2110.11837)

reflected light

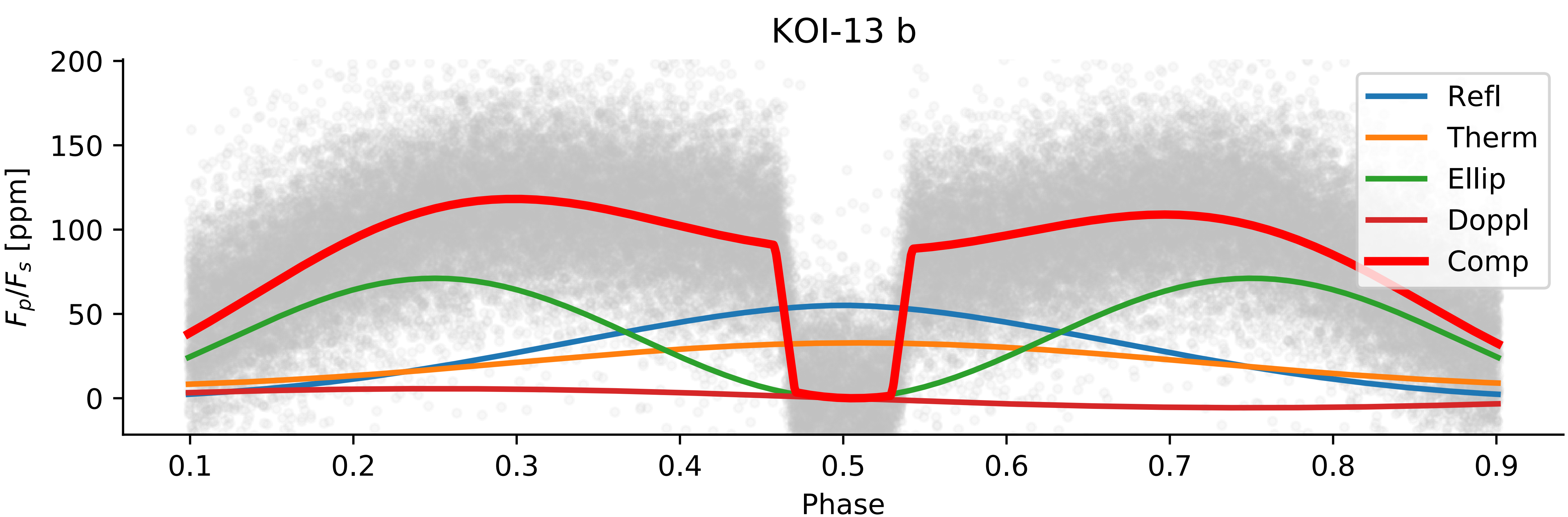
Uses novel analytical solutions of *Heng, Morris & Kitzmann (2021)* to fit for fundamental scattering parameters; obtains geometric and spherical albedos during post-processing

Code originally written by Brett Morris
(https://github.com/bmorris3/kelp)

What is kelp?

Open-source code to fit thermal and reflected-light components (and more) of phase curve using physically-motivated models.

Code originally written by Brett Morris
(https://github.com/bmorris3/kelp)



Preliminary! (Morris et al. 2022, in prep)